SYSTOLIC GEOMETRY AND 3D PRINTING

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The SMART Program

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PROJECT BACKGROUND AND GOALS

Background Information

The systole (sys) is defined as the length of the shortest loop on a surface or an object that cannot be contracted to a point. The sys is closely related to the surface area of an object. The systolic ratio (SR) is defined as Area/sys^2, of 2-D manifolds.

Project Goals

- Prove the Loewner's torus inequality: the SR of a torus is bounded from below by $\sqrt{3}/2$

  $SR(T^2) \geq \sqrt{3}/2$

- Learn 3D design and 3D printing

- Show the importance between theoretical mathematics and physical applications.

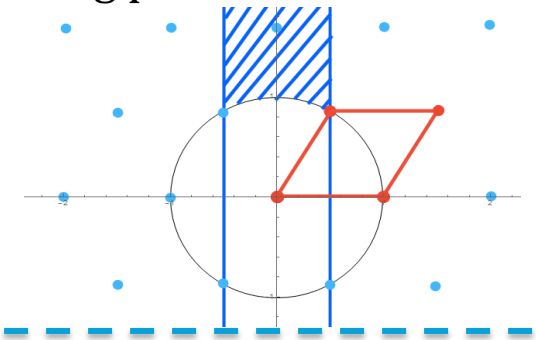
Why is it important?

Understanding the relationships between curves and surfaces is important for architecture design, physics, space packing, robotics, DNA structures and many other fields. For example, we can find ways to efficiently put more objects in a box after studying the curvature and surface area of the objects.

PROJECT RESULTS

SYSTOLIC GEOMETRY

- We studied *Elementary Differential Geometry 2nd edition* by Pressley, A.
- We learned and proved Loewner's Torus Inequality using point lattice:

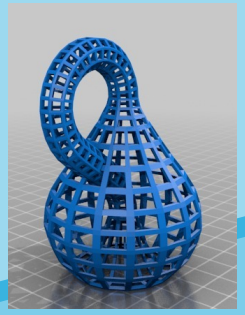


3D PRINTING

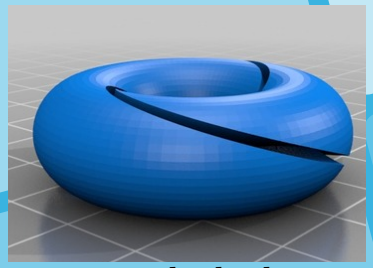
- We learned MeshMixer, an application for 3D design
- We printed a Klein bottle, and an interlocked torus using a 3D printer

IMPORTANCE

- We are still at a developing level of 3D printing. Understanding SR, the relationships between surface area and distances, makes it possible for us to better design 3D objects.
- SR is important for architecture design. Every shape has its curvature and other intrinsic values. Studying SR helps us better figure out the relationships between surface area, materials, gravity and the shape, and can thus design safer and more creative structures.
- 3D printing is the future. It is necessary for everyone to learn more about it.



Klein Bottle



Interlocked Torus

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